1. Abstract

According to an individual goal, risk tolerance, and investment horizon, the asset allocation is indicated to apportion a portfolio's asset to balance risks and rewards. Furthermore, it has been optimized to be determined primarily based on the asset classes, where three main asset classes with different levels of risk and return are normally defined as equities, fixed-income, and cash and equivalents, and each of them will behave differently during the investment period so enables investors to diversely allocate their assets with the selection of individual securities. To be specific, this approach targets to mitigate overall portfolio volatility by combining asset classes with low correlations and thus different moving directions at the same time, which together building an expected estimation of stable returns over time. Recently however, another approach is announced with a new type of building block as risk factors, which addresses the underlying risk exposures that drive the return of an asset class. To compare and evaluate these two approaches, both strategic asset allocation settings and tactical asset allocation settings, where the superior returns separately come from a granular opportunity set coupled with the long-term capital market assumptions and the one with the short-term capital market assumptions, will be considered to ensure no risk-adjusted returns are generated.

In addition, the paper aims to show that neither approach is inherently superior to the other, and it provides a historical view of how the risk factors being advanced, a mathematical proof, and optimizations using an empirical example based on different periods of historical data. The detailed contents will be narrated in the following sections and finally a brief conclusion will be placed with several takeaways regarding the individual thoughts generated after reading this article.

2. Background & Motivation

2.1 History of Risk Factors

To better understand the difference between asset-class-based asset allocation and factor-based asset allocation, an issue may arise as how the risk-factor-based approach being taken out; or in other words, why the traditional asset-class diversification is not enough when considering the asset allocation? To answer this question, the history of the risk-factor-based model should be discussed.

In 1950s, Harry Markowitz firstly worked on the mean-variance optimization for creating portfolios, and the asset-class based approach was implemented due to the hardness of running a single-stage optimization including all available individual securities. Since the mean-variance theory assumes that a portfolio is a weighted array of assets, the return of it should also be a random variable with an expected value and a variance while the standard deviation represents the risk existing. Based on this assumption, the market risk and the correlation of products when concerning about the target asset allocation can be analyzed. Nevertheless, it is necessary, yet impossible, to estimate all the correlations using time-series data as thousands of individual securities are involved. In this case, the structural multifactor risk model came up in 1970s and had been developed in following years. Concretely, the Sharpe-Lintner-Mossin-Treynor capital asset pricing model focuses on the market risk, and Fama and French extend this idea by supplementing the market factor with its size and valuation factors. Then, manager performances and systematic passive exposures are also brought into consideration with the seven-factor model being taken out in order to explain the hedge fund returns. To sum up, by constructing portfolios based on risk factors rather than the asset classes, managers can potentially build more efficient portfolios that require less risks to achieve competitive returns, while the overexposure to a particular risk factor within the portfolio is also capable to be detected and minimized. From this perspective, it may state that an investor should be a fan of risk-factor-based model like the authors of this paper, but it still argued whether the risk models are inherently superior over the asset-class-based ones.

2.2 Doubts toward the superiority of risk models

The main reason for doubting the superiority of risk models shown in many papers is the improper apples-to-oranges comparisons used in these papers. To be specific, the superior facts result form the comparison made between a simple asset-class set and a risk-factor set which contains more potential exposures. Obviously, for example, comparing a simply long-only stock portfolio with a various-factor set including valuation, momentum, and currency etc. that allows to go long or short can not obtain a fair conclusion.

Furthermore, it is impossible for all investors to accept the risk-factor-based asset allocation, as individual securities often belong to multiple factors and the offsetting position required tend to be easily influenced by the diversified market situations.

3. Idealized Proof

Next, the mathematically proof is applied towards the objective of this paper demonstrating that the solutions to two unconstrained mean-variance optimizations are equivalent, for one in asset-class space and the other is risk-factor based. Here, two assumptions are made to simplified model, which are firstly the number of factors equals to the number of assets and then the asset-class returns and the risk factors are mutually determined. To emphasized here, mean-variance optimization, expressed as variance against expected return, assumes that investors will make rational decisions with a seek for low risk and high reward. Furthermore, long-only constraint that has different implication treating the opportunity sets of asset classes and risk factors are set aside. By construction, many risk factors involve leverage that led to a comparable long-only optimization of risk factors in which the optimizer is not allowed to short is, in fact, less constrained than a long-only optimization of asset classes. Additionally, it can be generated that an advocate of the risk-factor-based model will not impose a long-only constrain on the asset-class optimization due to the investor preferences being reflected by most optimization constraints.

With the mean-variance analysis and the goal to find that there is no inherent advantage in the risk factors and returns for either approach, the variables of the realized returns, expected returns, and covariance matrices for both asset classes and risk factors. In this simplified model, the vector of asset-class returns is a linear transformation of the factors based on matrix L, where the asset class to risk factor exposure-mapping matrix L is square and invertible. In this case, we can gain the equations representing the optimal weights and a risk aversion coefficient is introduced as well. With the calculation shown in *Appendix 1*, for the returns of the asset-class-based and factor-based optimal portfolios, the risks, and the asset-class covariance matrix that can be derived from the risk-factor one.

4. Real World Optimization

4.1 Empirical Example

After the mathematical proof, a question just be raised as will the result found fit the situation in the messy real world? To find out the answer to this issue, two opportunity sets are utilized that more or less cover the same underlying universe of individual securities.

Appendix 2 shows annualized total return and excess return arithmetic means and standard deviations based on the historical data from the January 1979 to the end of 2011. The figures suggest that the average pairwise correlation for the risk factors is considerably lower than that for the asset classes. This correlation difference should be intuitive as the asset classes are all part of the market portfolio and, hence, carry with them an element of overall market risk.

4.2 Comparison

Then an optimization comparison is applied with a constraint set as the weight of the asset classes and of the risk factors to be nonnegative and the allocations summed up as 1. Because all the risk factors are derived series obtained by subtracting one series from another, it states that an investment of one dollar financed in the risk factor is seemed as the combination of investing in Treasure bills plus the self-financing risk factor. To visualize the annualized standard deviation using different time period data, an interesting point is found. The arithmetic annualized returns (*Appendix 3*) indicate that long-only asset based on the asset-class allocation appears to have an efficient frontier dominating the one based on the risk-factor approach. However, when considering the data that in a shorter and more current time period, neither asset classes nor risk factors consistently dominated along the whole risk spectrum. Thus, with the real-world optimization using the historical data, both approaches perform a superiority over the other regarding different time period given.

5. Conclusion & Takeaways

To conclude, mathematical proof given along with the empirical test based on the historical data suggests that neither approach is inherently superior to the other, and the particular superiority may appear due to different time period and specific composition of the factors. Thus, investors choosing different approach to allocate their assets need to be aware of the rather extreme positions embedded in the factors, especially for the risk-factor-based approach as the long-only constraint is dismissed in this paper which led to the lack of forward-looking macroeconomic view on a wide range of variables such as monetary policy, geopolitical developments, inflation, and interest rates. Furthermore, leverage contained in the risk-factor model may decrease of the acceptance among investors.

Appendix

Appendix 1

The detailed calculation of the mathematical proof.

$\mathbf{r}_a, \boldsymbol{\mu}_a, \boldsymbol{\Sigma}_a, \mathbf{r}_f, \boldsymbol{\mu}_f$, and $\boldsymbol{\Sigma}_f$

$$\mathbf{r}_a = \mathbf{L}\mathbf{r}_f, \quad (1a)$$

and

$$\mathbf{r}_f = \mathbf{L}^{-1}\mathbf{r}_a. \quad (1b)$$

$$\boldsymbol{\Sigma}_a = \mathbf{L}\boldsymbol{\Sigma}_f\mathbf{L}', \quad (2a)$$

and

$$\boldsymbol{\Sigma}_f = \mathbf{L}^{-1}\boldsymbol{\Sigma}_a(\mathbf{L}')^{-1}. \quad (2b)$$

$$\mathbf{w} = \frac{1}{\theta}\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}, \quad (3)$$

where $\mathbf{w}$ represents the optimal weights and θ is the risk aversion coefficient. Notice that we have removed the subscripts because the solution can be calculated using either asset classes or risk factors. After calculating the optimal weights in one space, we can move to the other space using

$$\mathbf{w}_a = (\mathbf{L}')^{-1}\mathbf{w}_f \quad (4a)$$

and

$$\mathbf{w}_f = \mathbf{L}'\mathbf{w}_a. \quad (4b)$$

Appendix 2

Table 1. Historical Returns and Standard Deviations, January 1979–December 2011

	Total Return		Excess Return	
Asset Class/Factor	Arithmetic Mean	Standard Deviation	Arithmetic Mean	Standard Deviation
Barclays U.S. Treasury TR	8.59%*	6.12%*	2.96%	5.79%
Barclays U.S. credit TR	9.05*	7.87*	3.40	7.50
Barclays U.S. MBS TR	8.82*	7.40*	3.18	7.02
Russell 1000 Growth TR	12.22*	19.92*	6.42	18.97
Russell 1000 Value TR	13.04*	16.97*	7.20	16.15
Russell 2000 Growth TR	12.18*	26.47*	6.38	25.24
Russell 2000 Value TR	14.86*	20.40*	8.93	19.44
Market	12.70**	17.85**	6.88	17.00
Size	6.29**	11.02**	0.77	10.49
Valuation	6.44**	10.24**	0.91	9.68
Term spread (duration)	10.64**	13.44**	4.91	12.83
Credit spread	5.93**	3.96**	0.42	3.73
Mortgage spread	5.71**	3.97**	0.22	3.66
Citigroup 3-month Treasury bills	5.48	1.04	0.00	0.00

Note: TR stands for total return.
*Historical capital market assumptions for the asset classes being optimized.
**Historical capital market assumptions for the risk factors being optimized.

Appendix 3

Figure 1. Long-Only Constraints: Asset Classes vs. Classic Risk Factors, January 1979–December 2011

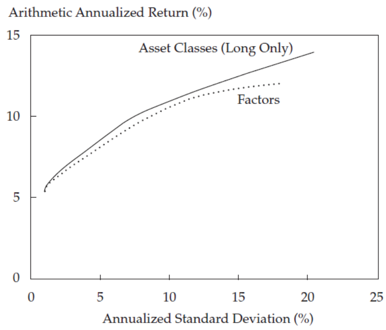

Figure 2. Long-Only Constraints: Asset Classes vs. Classic Risk Factors, January 2002–December 2011